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On Plasma Sheath Resonant Energy Absorption in Collisionless Plasmas

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14. ABSTRACT

We have performed experiments designed to investigate the radiation characteristics of a spherical capacitive probe in a plasma environment in the large Space Physics Simulation Chamber at the Naval Research Laboratory. In the process we are able to approximate plasma density and electron neutral collision frequency in simulated space plasma environments consistent with earlier experimental efforts using plasma impedance probes. By using only the S_{11}-parameter outputs of a network analyzer, or the reflection coefficients, we are able to unfold both these quantities with measured data sets. In addition, we observe significant energy absorption at frequencies much less than the plasma frequency which we associate with a sheath-plasma resonance. This report is an exposition of this method along with data results and a comparison to theory.

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Abstract

We have performed experiments designed to investigate the radiation characteristics of a spherical capacitive probe in a plasma environment in the large Space Physics Simulation Chamber at the Naval Research Laboratory. In the process we are able to approximate plasma density and electron neutral collision frequency in simulated space plasma environments consistent with earlier experimental efforts using plasma impedance probes. By using only the S_{11}-parameter outputs of a network analyzer, or the reflection coefficients, we are able to unfold both these quantities with measured data sets. In addition, we observe significant energy absorption at frequencies much less than the plasma frequency which we associate with a sheath-plasma resonance. This paper is an exposition of this method along with data results and a comparison to theory.

Introduction

The use of an RF impedance probe to measure plasma parameters is a well-established experimental method investigated earller in both the laboratory *[Uramoto, 1970]* and in the space plasma environment *[Jackson and Kane, 1959]*. More recently in space, *Abdu et al., [1991]* used a two frequency High Frequency Capacitive probe to investigate electron density and temperature associated with ionospheric bubbles under conditions of equatorial spread-F. Also, *Jensen and Baker, [1992]* demonstrated the range of the Plasma Frequency Probe (PFP) technique by measuring electron density as low as 7 x 10^3 cm^{-3} during a nighttime rocket ascent to an apogee of 450 km in a study of thunderstorm formation. Theoretical treatment and the analysis of data from impedance probes of various shapes and sizes has been treated in numerous studies related to space plasmas, eg., *[Balmain, 1964; Nikitin and Swenson, 2001]*. There is a body of early theoretical and experimental work related to the phenomenon of RF resonance both above and below the plasma frequency *[Harp and Crawford, 1964; Dote and Ichimiya, 1964]*

The basis of these measurements lies in the observation that the impedance properties of an antenna or probe are altered by the presence of a plasma and from these alterations plasma characteristics can be determined. In vacuum, for example, there is no significant electromagnetic

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radiation from a loop antenna until the wavelength of the driving signal is on the order of the loop circumference. Antenna size, along with its shape, with respect to frequency influences power requirements necessary to produce detectable electromagnetic radiation. In this case, a radiation resistance is defined by the ratio of input power to the square of the antenna current. When power is being radiated and there is insignificant internal antenna resistance this ratio allows a definition of a "resistance" and hence a radiated power. A device measuring the ratio of input power to reflected power from such an antenna would see this ratio drop to less than one assuming there is no impedance mismatch between the signal generator and the antenna with its associated cabling. In the presence of a plasma, however, there is an additional means to transfer energy from the antenna through internal plasma currents which can arise due to the presence of antenna current itself and the interaction of the antenna with the plasma which occurs through the sheath. Measuring antenna impedance when the antenna is placed inside a plasma then provides a method of studying the plasma oscillations which are excited and from this study to infer plasma characteristics.

Langmuir oscillations in a cold plasma occur at the plasma frequency and are expressible in terms of an impedance which is only capacitive and inductive. However, if there are collisional processes associated with the plasma currents the impedance acquires a resistive component. And so it is natural to associate this energy transfer also with an effective antenna resistance in the same manner as is done in the classical definition of radiation resistance cited above. At first glance in a collisionless plasma there would appear to be no means of dissipating the driving energy to extract useful energy (again assuming negligible antenna resistance) since there is no resistive term and the capacitive and inductive responses are adiabatic. Nevertheless, energy absorption often occurs in such plasmas for various antenna shapes immersed in the plasma and for which there are plasma sheath formations. . It has long been known that energy dissipation can occur at frequencies lower than the plasma frequency in collisionless plasmas through the phenomenon of sheath-plasma resonance *[Takayama et al. 1960; Barston, 1964; Balmain, 1964; Harp and Crawford, 1964; Buckley, 1966; Crawford and Harker, 1972; Stenzel, 1988; Ku et al., 1998(1); Ku et al., 1998 (2); Tanizuka and Allen; 1999]*.

In the simplest model of the antenna immersed in a plasma the sheath is treated as a vacuum capacitor and the plasma as a series combination of inductor-resistor in parallel with a capacitor *[Blackwell, et al., 2004]*. There are then parallel and series resonant solutions for such a circuit when driven by an AC signal. For a particular frequency, resonances occur which fundamentally result from the fact that at resonance the reactance of the antenna-plasma system is near zero and therefore the impedance becomes all resistive *[Messian and Vandenplas, 1967]*. To more accurately predict the result of applying an AC signal to a probe immersed in a collisionless plasma , the simple hydrodynamic approach must be replaced by a solution of the Vlasov equation *[Crawford and Harker, 1972; Buckley, 1966]*. In this case plasma resonances have been shown to be related to gradients in electron density within a non-uniform plasma bounded by sheaths and described above as a plasma capacitor. The resonances are "Landau-type" *[Bekefi, 1966]* resistive components arising in cold, collisionless plasmas. Toward a more physical insight into these resonances, it was pointed out in a brief communication by *Buneman, [1961]* that a network of a large number of resonating circuits with differing time constants exhibits resistance properties as a transient response. From a physical point

of view he argues that this simply represents a different form of energy dispersal than in the collisional case. Since there are a multitude of ways that the energy can be dispersed, the process is thermodynamically irreversible in the sense that it is highly unlikely that the energy would be returned in its original form. There is a clear analogy here to a plasma with density gradients if one pictures this plasma a composed of discreet circuits as described above with varying resonant frequencies.

Our aim in the brief work here is to present preliminary experimental results which indicate energy absorption in a collisionless laboratory plasma at frequencies less than the plasma frequency. The measurement method uses only the ratio of incident to reflected power for a small Aluminum sphere inserted into a plasma and driven by an RF signal. In an accompanying paper we have used the basic circuit model described below in order to understand impedance observations obtained from the network analyzer for the driven sphere. In addition that paper shows how one can infer both plasma density and sheath thickness from the measurements. In this paper, we concentrate on observations of energy absorption and approximations to an "effective" collision frequency. Section I is a brief overview of theory and Section II is a presentation of data and a comparison to predictions of the simple model used.

I) Theory

The simplest approach to an investigation of the effect of applying an rf field to a metallic object inside a vacuum chamber and immersed in a plasma is to assume that the configuration behaves approximately as a capacitor with a dielectric *[Blackwell et al., 2004]*. The boundaries are the conductor surface and the vacuum chamber walls. For an ordinary dielectric $C = C_0 k$, with k the dielectric constant where $k>1$ and C_0 is the vacuum capacitance. For the plasma capacitor, $C = C_0 \epsilon(\omega)$ where $\epsilon(\omega)$ is the dielectric constant, or permittivity. The simplest estimate for $\epsilon(\omega)$ is based on Maxwell's wave equations for a cold, non-drifting, unmagnetized plasma,

$$\nabla x(\nabla x\boldsymbol{E}) = -\frac{1}{c^2}\frac{\partial}{\partial t}(4\pi\boldsymbol{j}+\frac{\partial \boldsymbol{E}}{\partial t}) \equiv -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\epsilon\boldsymbol{E}) \tag{1}$$

In this form **E** is the electric field and **j** is current density. Since electrons are the lightest species they carry most current and so we have that,

$$\boldsymbol{j}=en_e\boldsymbol{V_e} \tag{2}$$

where e is charge, n_e is density and V_e is the mean velocity of plasma electrons. The mean velocity is determined from the electron momentum equation and with no magnetic field this is,

$$\frac{\partial}{\partial t}(n_e m V_e)+\nabla\cdot(n_e m V_e V_e)+\nabla(n_e T_e)=en_e \boldsymbol{E}-\nu_{en} n_e m \boldsymbol{V}_e \tag{3}$$

where m is mass, T_e is temperature and ν_{en} is the electron-neutral collision frequency. We assume a non-drifting plasma without gradients and so we can ignore the second (convective) and third (pressure) terms on the left side of Eqn (3). The equation is then linear and we assume a Fourier solution of the form $e^{-i\omega t}$ for $\mathbf{V}_e$ and $\mathbf{E}$. For this case we find,

$$\boldsymbol{j}=\frac{e^2 n_e}{m(\nu_{en}-i\omega)}\boldsymbol{E}=\frac{\omega_p^{\,2}}{4\pi(\nu_{en}-i\omega)}\boldsymbol{E} \tag{4}$$

where ω_p is the electron plasma frequency. In this simplified approximation the dielectric constant which characterizes the plasma becomes,

$$\epsilon(\omega) \;=\; 1+i\frac{\omega_p^{\,2}}{\omega(\nu-i\omega)} \;=\; \frac{\omega^2+\nu_{en}^2-\omega_p^2}{\omega^2+\nu_{en}^2}+i\frac{\nu_{en}\omega_p^2}{\omega(\omega^2+\nu_{en}^2)} \tag{5}$$

Resistive effects are seen then to contribute via the imaginary part of $\epsilon(\omega)$. Ion and neutral collisions both contribute to a collision frequency but ν_{en}, the electron-neutral collision frequency, is dominant over all other contributions under conditions of weak ionization or when $\nu_{en}/N < 10^{-3}$ where N is neutral gas density. In the experiment here we use an approximation to electron-neutral collision frequency as $\nu_{en} = 7 \times 10^{-8}\, N\, T_e^{1/2}$ and a neutral pressure of 2×10^{-4} Torr ($N \sim 6.6 \times 10^{12}$ cm^{-3}). Electron density is typically on the order of 10^8 cm^{-3}.

It was noticed early on that if one uses the simple form for the permittivity in Eqn (5), the impedance for the plasma capacitor, C, has the form of the impedance of a series resistor and inductor, in parallel with a capacitor, C_0, i.e., rearranging Eqn (5) and substituting for C we have,

$$Z=\frac{1}{i\omega C}=\frac{1}{i\omega\epsilon_r C_0}=\frac{1}{[i\omega C_0+[\frac{\nu}{\omega_p^2 C_0}+i\frac{\omega}{\omega_p^2 C_0}]^{-1}]} \tag{6}$$

where this familiar form of the impedance allows us by analogy to define the circuit inductance and resistance as *[Lieberman and Lichtenberg, 1999; Blackwell et al., 2004]*,

$$L_p=\frac{1}{\omega_p^2 C_0},\quad R_p=\frac{\nu}{\omega_p^2 C_0}=\nu L_p \tag{7}$$

where ν is taken as the total collision frequency. This idea leaves open the possibility of interpreting ν of Eqn (7) loosely as an "effective" collision frequency and we will discuss collisionless absorption through this simple model and medium. A radiation "resistance" is then related to the plasma inductance as seen in Eqn (7).

The idea expressed by *Buneman [1961]* and put forth above has been used in a number of studies relating the "effective" resistance (through the analogy of a number of different resonant circuits) to plasma density gradients. This association with resistance in one instance arises from first calculating the power absorbed by the plasma in a collisionless Vlasov calculation and comparing that to the power delivered *[Crawford and Harker, 1972]*. The expression for the power is a function of the parallel plate capacitor configuration and the density profile and is not generally applicable to other geometries. The results are nevertheless useful for estimating energy absorption. The time-averaged power input into a parallel plate capacitor of area A, which is shown also to be absorbed at the resonant layer, is given by,

$$\langle P \rangle = \frac{\pi \omega^2}{\left| \frac{d\omega_{pe}}{dx} \right|} \left(\epsilon_0 \frac{\hat{E}^2}{2} \right) A \tag{8}$$

where $j = i\omega\epsilon_0\hat{E}$ with $\hat{E}$ related to the perturbation electric field. and the derivative is taken at the resonant layer position. In a collisionless plasma, if we use displacement current in place of real current, the instantaneous power absorbed for the parallel plate geometry is simply,

$$P = I^2 R = \left(\epsilon_0 \frac{\partial \Phi_E}{\partial t} \right)^2 R \cong \epsilon_0^2 \omega^2 A^2 \hat{E}^2 R \tag{9}$$

with the time averaged total power given as,

$$\langle P \rangle = \frac{1}{T} \int_0^T I^2 R \, dt = \frac{1}{T} \int_0^T \left(\epsilon_0 \frac{\partial \Phi_E}{\partial t} \right)^2 R = \frac{\epsilon_0^2 \omega^2 A^2 \hat{E}^2 R}{2} \tag{10}$$

Comparing Eqns (8) and (10), for the case of the cold, collisionless plasma slab, the effective resistive component appears as *[Tanizuka and Allen, 1999]*,

$$R = \frac{\pi}{\epsilon_0 A \left| \frac{d\omega_{pe}}{dx} \right|} \tag{11}$$

where the derivative of the plasma frequency is evaluated at the plane of local resonance and A is the area of the probe. We are investigating the implications of assigning R of Eqn (11) as an effective resistance and present below some first efforts in this regard.

II) Data and Comparison to Theory
a) Energy Absorption

We plot in Figure 1 an example of the energy absorption for the small sphere compared to theoretical estimates using the simple theory above. The data is taken for a plasma density of 2.5 x 10^8 cm^{-3} and a neutral pressure of 3.4 x 10^{-4} Torr. The abscissa of the plot is normalized driving frequency and the ordinate is reflected power divided by the incident, $|\Gamma|^2$,(or $|S_{11}|^2$ as is the designation for power measured by the network analyzer). Recall that Γ is defined as the ratio of the amplitude of the reflected signal to the incident and is in general complex. From Γ one can calculate the impedance of the medium with which energy is exchanged or deposited.

The theoretical curves are plotted for 3 values of effective collision frequency all of which are at least two orders of magnitude higher than the actual collision frequency based on electron-neutral collisions. The theoretical curves were calculated using the impedance of Eqn (6) and the assumption of 50 ohm impedance matching between the cabling and the network analyser.

What is clear from this plot is that if we are to associate the apparent energy absorption seen with collisonless absorption, we must find that the effective collision frequency is much larger than the usual resistive component assumed responsible for the energy exchange. Below we use the argument *[Harker and Crawford, 1972]* outlined above to approximate an effective resistance responsible for this which is based on a plasma density gradient. This could account for the difference between theory and experiment as seen in Figure 1, i.e., our assumption in deriving the expression for the permittivity seen Eqn (5) included the absence of density or temperature gradients in the plasma and hence the neglect of any pressure terms.

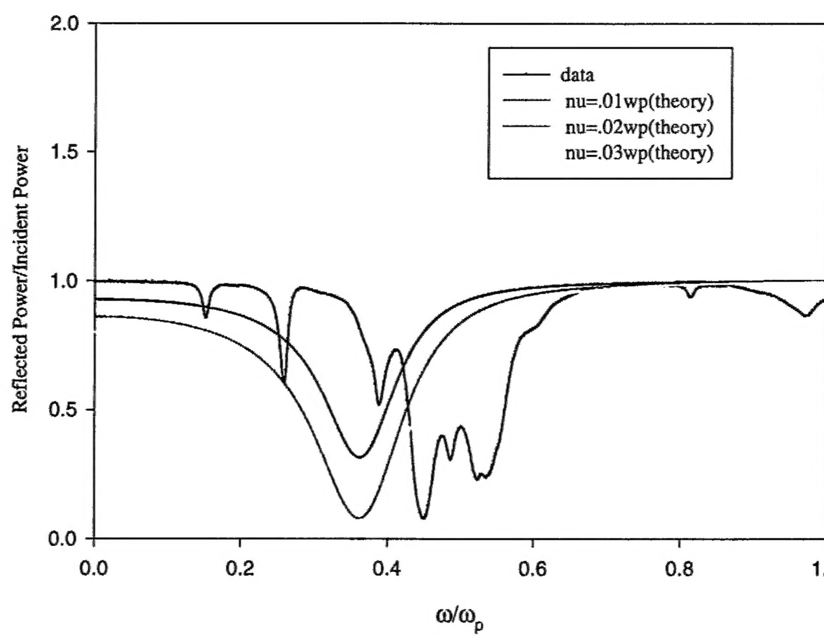

Figure 1 - The ratio of reflected to incident power as measured by a spectrum analyser: black-Experiment; color-theory

b)Absorption as a function of Debye length

Figure 2 is a plot of the normalized frequency of the resonances observed as a function of Debye length. The data in Figure 2 was gotten by finding the maximum power absorption as a function of normalized frequency and then plotting versus a normalized probe radius for data sets such as presented in Figure 1. The plasma density was varied to produce the normalized probe radius. For example, the radius of the Aluminum sphere is on the order of 1 cm and for $\lambda_D \sim 0.05$ cm , $n_e \sim 2.5 \times 10^8$ cm^{-3}, and $R_p/\lambda_D \sim 20$ and for this case we are in the range where $R_p >> \lambda_D$ as can be seen from the abscissa values in the figure. The theoretical curve in Figure 2 is taken from calculations by *Harp, [1965]* and a further treatment and exposition of the issue by *Bekefi, [1969].* According to these calculations in the quasi-static approximation for monopole excitation of a sphere, the resonance positions are given approximately by,

$$\omega_r = \omega_p \sqrt{\frac{s/R}{1+s/R}} \tag{12}$$

where s is the sheath length. Further calculations using the Vlasov equation *[Pavokovich and Kino, 1963; Harp, 1964]* concluded that an approximation to s valid in many regimes is $s \sim 5\,\lambda_D$, *[Bekefi, 1969].* This approximation is shown to be valid as long as R is not too small compared to λ_D. Using this value for s for the theoretical part of Figure 2, it appears that for our case the approximation is also a close one. The electron densities used in the calculation of λ_D were measured by a conventional Langmuir probe. The resonance frequencies, which were selected at the position of maximum energy absorption as described above, were taken without regard to the frequency width of the resonance and so there are no resolution estimates on the data.. From Figure 1 it can be seen that the minima are very well defined. We conclude from this figure that the calculations based on sheath plasma resonance positions as in Eqn (12) above are very close to the experimental data and so this is another indication that the interpretation of the data based on this scenario is accurate.

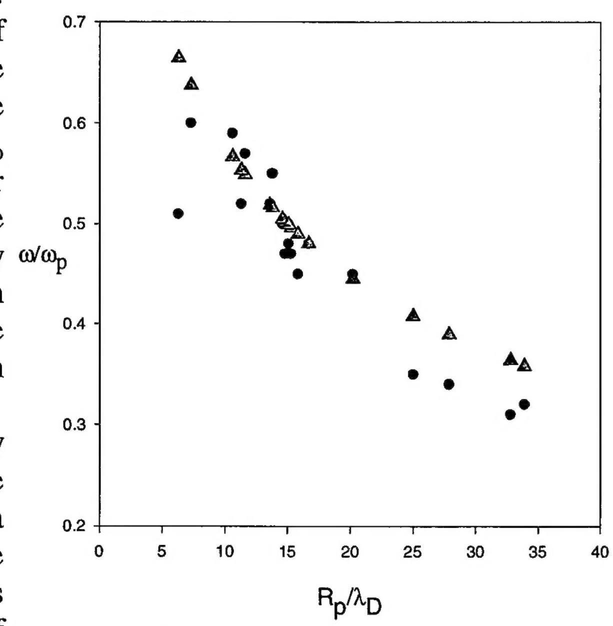

Figure 2 - Resonant frequencies vs Debye length black (data); green (theory)

c) Effective collision frequency

The expression for time-averaged power seen in Eqn (8) was derived under the assumption of a parallel plate capacitor. Using the expression for effective plasma resistance seen through Eqn (11) we may estimate the area, A, for a parallel plate capacitor as a function of effective collision frequency given our measured density gradient at the approximate position of the probe. ie., substituting for R in Eqn (12) we find,

$$A = \frac{\pi}{\epsilon_0 \nu_{eff} L_p \left| \frac{d\omega_{pe}}{dx} \right|} \tag{13}$$

Figure 3 is a plot of this area as a function of ν_{eff} and includes the range covered in Figure 1, i.e., ,01 $\leq \nu_{eff} \leq$ .03. These area estimations are in the general range of the vacuum chamber size, eg., assuming a spherical chamber region whose radius is the actual cylindrical chamber radius, R = 0.9 m, requires $\nu_{eff} \sim .045$. In order to compare our observations to a model of this nature more realistically it is necessary to provide the appropriate form of Eqn (11) for spherical geometry. This work is currently in progress.

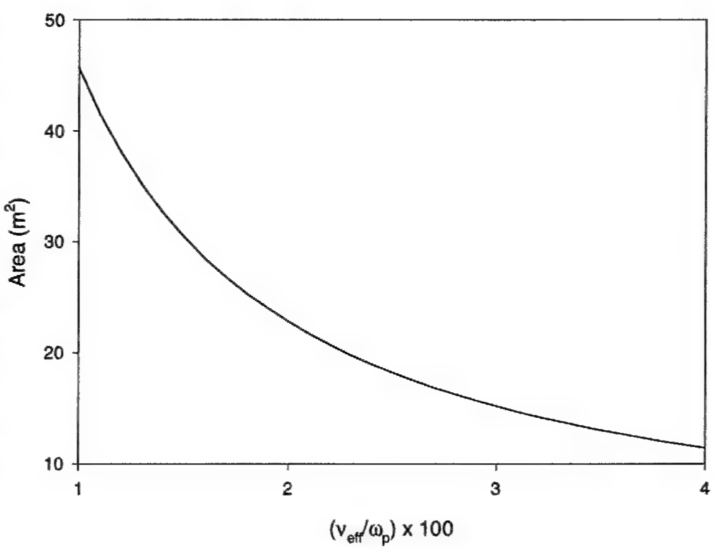

Figure 3 - Area versus normalized collision frequency for the parallel plate capacitor as derived from effective resistance

d) Chamber radial density variation

Shown in Figure 4 is the plasma frequency, ω_{pe}, as a function of radial position taken from electron density measured by a Langmuir probe. Figure 4 was used to determine the approximate value of the plasma density gradient used in Eqn (13) through a linear fit to the data from r = 0 to r = -.25 m (Note: although designating r as a negative number is strictly not correct, it is clear from this usage and the figure what is meant for a radial traversal). Each data point is the average of a number of trials at a given radius and the spread in individual measurements is no greater than 10%.

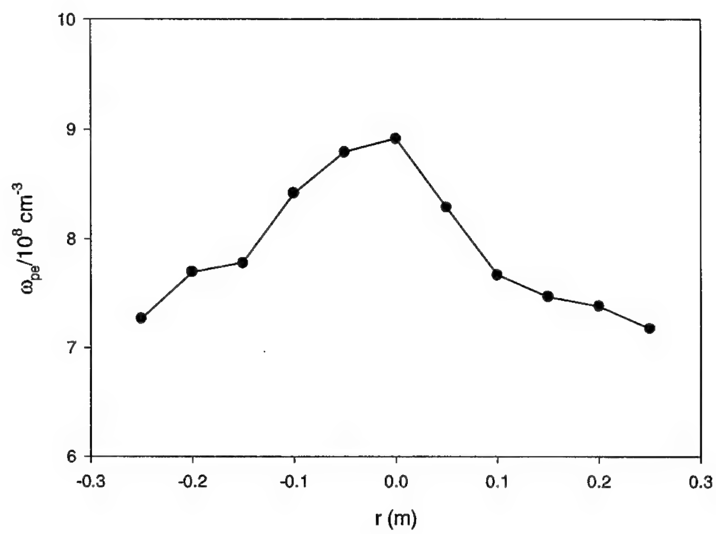

Figure 4 - Radial plasma density variation in the experimental area

e) Magnetic field dependence

The presence of a magnetic field is expected to drastically alter the shape of the sheath with respect to the field direction in addition to a multitude of other possible effects *[Bell and Wang, 1971]*

and therefore conclusions based on the field free case are not expected to hold in general. However, in order to maintain our plasma, it is necessary to apply a small axial magnetic field on the order typically of ~ 2 Gauss. Shown in Figure 5 is a plot of the variation in resonance minimum as function of frequency and magnetic field varying from 1 to 10 Gauss. Although the range of variation of the magnetic field for this data is limited it nevertheless represents the actual range of magnetic field variation applicable in our experimental setup. Figure 6 is a plot of the maximum normalized positions of the resonance versus B as taken from Figure 5. Although there appears to be some effect discernible from the plot there is not a clear trend nor a dependence of peak position on magnetic field, at least for this data set in this limited range of B variation.

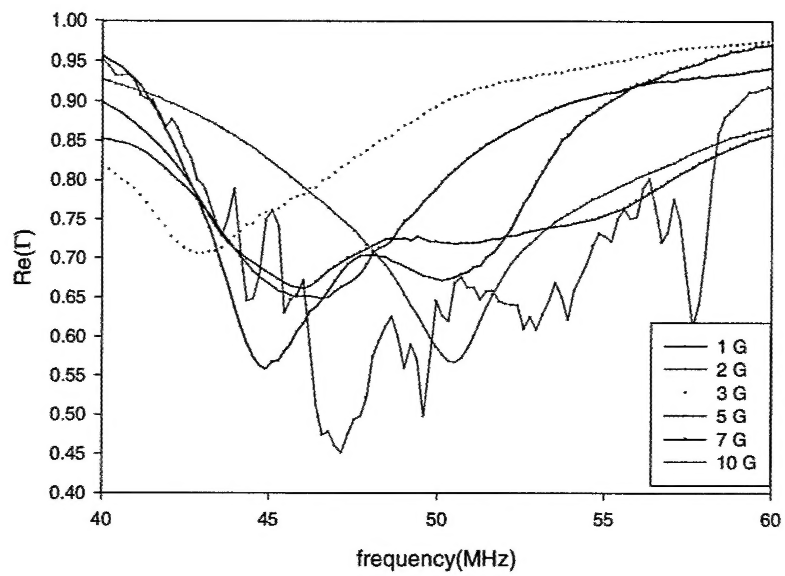

Figure 5 - Variation in resonance minimum for differing magnetic fields

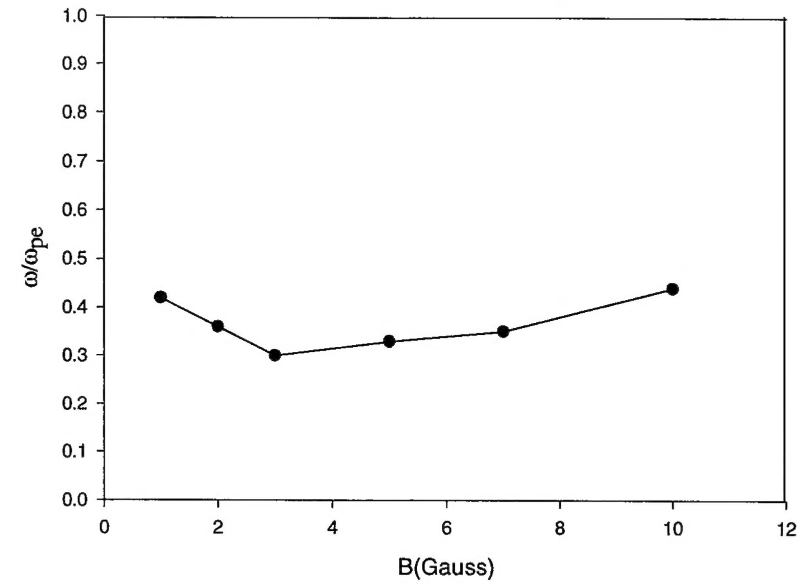

Figure 6 - Plot of variation of minima observed in Figure 5 versus magnetic field

Conclusions

We have observed collisonless energy absorption using a network analyzer for a small spherical probe in a vacuum chamber. Although this observation is consistent with the work of others we have compared as an order of magnitude estimate an earlier theoretical derivation of the effective plasma "resistance" for collisionless energy absorption. This calculation is based on a parallel plate capacitor and further work is needed to unfold the proper estimate for spherical geometry. We have estimated the effective collision frequency based on observations of reflected vs incident power necessary to produce the energy absorption observed. Our estimates of effective collision frequency are largely based on results of measuring the reflected versus incident power to the probe using a network analyzer. As in earlier investigations, the actual collision frequency based on electron-neutral collisions is at least 3 orders of magnitude lower than that necessary to produce the observed energy deposition. Finally, there does not appear to be a magnetic field dependence for the low ranges of B necessary to maintain our plasma.

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